The stress and strain fields in the neighbourhood of a notch in polyethylene

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(Received 1 October 1988; revised 22 November 1988; accepted 29 November 1988)

The stress and strain fields were determined in the neighbourhood of a sharp notch in a single edge-notched tensile specimen. The effects of notch opening, distance from the notch and unloading the specimen and the difference between plane stress and plane strain were investigated. From the measurements of the strain field, the stress was inferred directly from the stress–strain curve of the material. In general, the stress field is fairly uniform between the notch and the craze tip, in good conformity with the assumption of the Dugdale theory. Complexities in the strain field arise from the stress concentration at the corners of the bottom of the notch and from the characteristics of the craze.

(Keywords: polyethylene; fracture; crazing; stress field)

INTRODUCTION

Brittle fracture in polyethylene (PE) is initiated at a notch. There are two modes of brittle fracture in PE: (1) low-temperature fast fracture, and (2) slow crack growth at room temperature and above under low stresses. This paper is concerned with the zone of damage that emanates from a notch and initiates slow crack growth. Under plane-strain conditions the primary damage consists of a craze. Brown and Wang[1] have measured the strain field on the boundary of such crazes. Wang, Fager and Brown[2] found that the strain field in the neighbourhood of the notch was not only associated with the characteristics of the craze but was also determined by the concentration of stress produced by the corners at the bottom of the notch. The stress field connected with these strain fields was directly inferred[1,2] from the stress–strain curve of the material.

There have been many investigations of the zone of damage that emanates from a notch in polymers. Narisawa *et al.*[3–5] showed the effect of notch radius where a plastic zone forms and then a craze or crack occurs within the plastic zone. Wang and Kramer[6] measured the displacement profile of crazes and calculated the stress field by assuming the matrix was linear elastic. The method of determining the strain and stress fields in the present investigation is different in that the strain distribution in the neighbourhood of the notch and on the boundary of the craze is measured directly and the accompanying stress field is inferred from the tensile stress–strain curve of the material. One limitation of the present method is that the actual strain field is triaxial and only a single component of the strain is measured, and there is a weakness in that this strain is related to the stress by means of a tensile stress–strain curve instead of one that is more directly related to the triaxial state of stress. In spite of these weaknesses, the present method is an advance on existing methods, which are merely qualitative or determine the stress by assuming linear stress–strain behaviour for the material.

The specific aspects of the stress and strain fields in the neighbourhood of a notch that were investigated are: (1) the effect of varying the notch opening; (2) the effect of distance from the craze boundary; (3) the effect of unloading the specimen; and (4) a comparison of the damage zones produced under plane-strain and plane-stress conditions. These measurements of the stress and strain fields in the neighbourhood of the notch provide useful information for a quantitative understanding of the fracture process.

EXPERIMENTAL

The material was an ethylene–hexene copolymer with 4.5 butyl chains per 1000 carbons; density = 0.938, $M_n = 15\,000$ and $M_w = 170\,000$. This material has excellent resistance to slow crack growth and therefore is used for gas pipes. The damaged zones near the notch that were observed have the same characteristics as the damaged zone that occurs in gas pipes when they are in service.

The method for measuring the strain field has been described previously in detail[1]. In essence the damaged zone is produced under plane-strain conditions. A thin slice of the specimen is cut with a razor blade, which forms a parallel array of scratches. When the notch in the slice is reopened, the spacing between the scratches changes in accordance with the strain distribution. The spacing is measured from an SEM photograph of the scratches that was made while the notch was held open in the microscope. The strain can be measured with a precision of about $\pm 0.2\%$ strain.

The stress is inferred from the tensile stress–strain curves of the material as shown in *Figures 1a* and *1b* for small and large strains, respectively.

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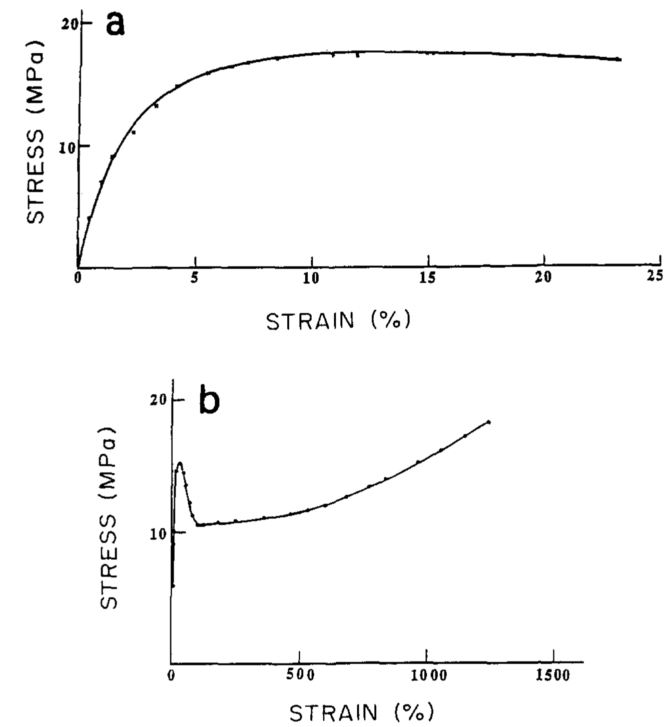

Figure 1 Stress–strain curves of ethylene–hexene copolymer: (a) small strains at 20°C and strain rate 0.04 min^{-1}; (b) large strains at 42°C and strain rate 0.4 min^{-1}

RESULTS

The SEM micrographs in *Figure 2* show the deformed area in the neighbourhood of a notch. The deformation was first produced in a single edge-notched tensile specimen under plane-strain conditions for a 2 mm deep notch with a constant stress of 4 MPa that was maintained for 106 min at 80°C. The resulting notch opening as measured at the bottom of the notch was 86 μm. A thin slice was taken from the centre of the 18 mm wide specimen and the notch was reopened to 86, 94, 116 and 138 μm at room temperature. The SEM pictures for each notch opening are shown in *Figures 2A–D*.

The main feature of the deformed area is the craze. This craze was originally formed during the exposure to the 4 MPa stress at 80°C. When the stress was first applied the dimensions of the craze closely corresponded to those predicted by the Dugdale theory, as shown by Lu and Brown[7]. After 106 min exposure to the 4 MPa stress, its size grew to that shown in *Figure 2A*. It is to be noted that opening the notch at room temperature to successively higher values of the notch opening did not appreciably change the length of the craze. When the notch was opened to 116 μm, as shown in *Figure 2C*, some fibrils were broken at the base of the craze.

Another prominent feature of the deformation is the dark areas near the corners at the bottom of the notch. These areas appear to consist primarily of shear deformation, but some microcrazes also form in these regions. These areas increase with the notch opening.

The other feature of the deformation is the strain that produces the variation in the spacing of the originally equidistant scratches, a quantitative assessment of the strain field near the notch and on the boundary of the craze has been obtained. The change in spacing of the scratches designated 0 and 1 (*Figure 2*) determined the strain. *Figure 3* shows this strain distribution along the boundary of the craze for the component of strain parallel to the applied stress. Curve A in *Figure 3* shows that the strain is nearly constant along the boundary of the craze except for a small maximum near the tip of the craze. As the notch opening was increased, the general level of strain increases, but the most prominent increase in strain occurs near the notch tip, as manifested in the highly deformed area shown in *Figure 2*.

The stress distributions (*Figure 4*) that correspond to the strain distributions in *Figure 3* have been obtained by using the stress–strain curve of *Figure 1a*. The outstanding feature of these stress distributions is that the stress is essentially constant along the craze, with the level increasing somewhat with increasing notch opening. The nearly constant stress along the craze is consistent with the Dugdale theory. The slight minimum in stress for the largest notch opening, which occurs near the notch tip, is associated with the decrease in stress that occurs

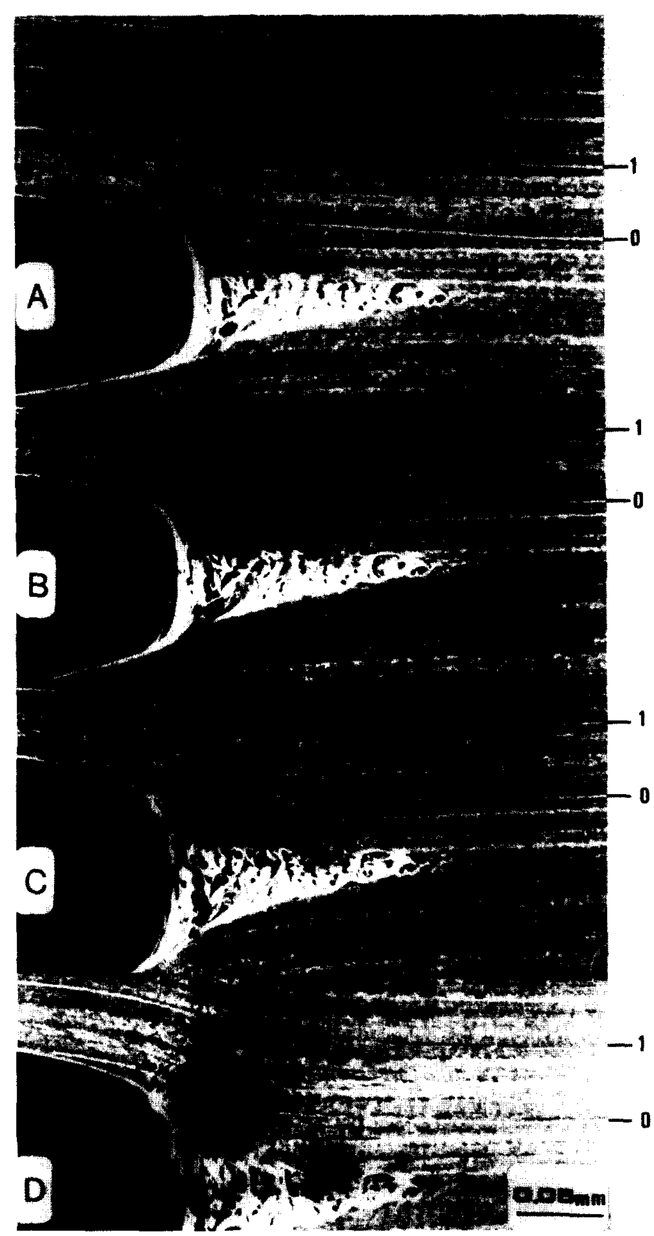

Figure 2 SEM micrographs for various notch openings. The notch length is 2 mm. The original craze was formed at 80°C under 4 MPa stress, which was kept for 106 min so that its notch opening was 86 μm. (A) 86 μm; (B) 194 μm; (C) 116 μm; and (D) 138 μm

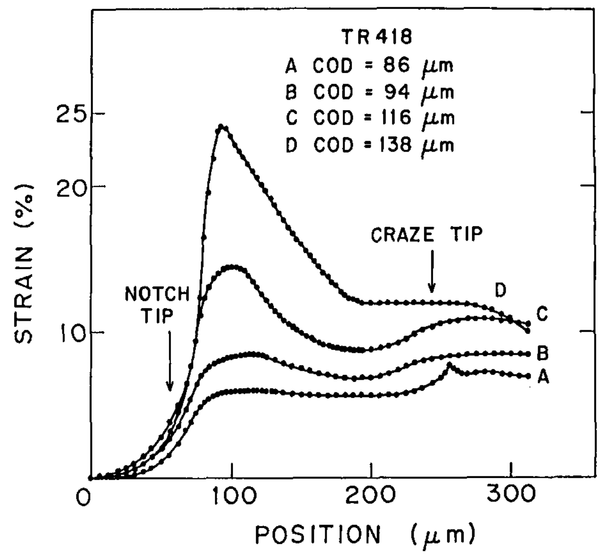

Figure 3 Strain *versus* position along notch and craze for the notch openings in *Figure 2*. The 0 position is the point where the strain becomes zero

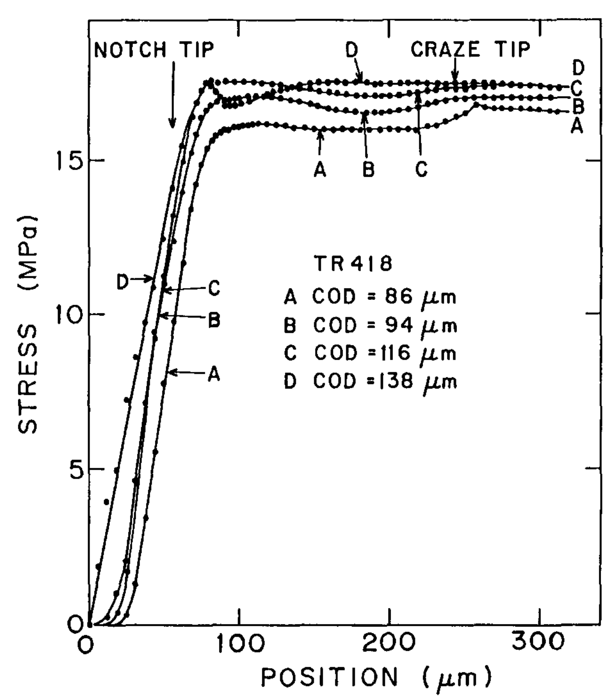

Figure 4 The stress distributions based on the strain distributions in *Figure 3* and from *Figure 1a*

beyond the yield point, as shown in *Figure 1b*. The stress distribution immediately to the left of the notch tip does not drop more rapidly to zero than it actually does because the strain that was measured is not exactly at the free surface.

There is some uncertainty as to whether the stress distributions presented in *Figure 4* actually represent the stress that exists while the specimen was loaded. Part of the uncertainty arises from the fact that the strain was not produced under a uniaxial stress but the stress was inferred from a uniaxial stress–strain curve. There is also the uncertainty associated with the relaxation in stress that occurs while the specimen was being photographed. In order to get another view of the stress distribution the residual strain was measured after the specimen was unloaded. The recovered strain was then determined and the stress prior to unloading was associated with the recovered strain by means of the stress–strain curve (*Figure 1a*).

Figure 5 shows the micrographs of the unloaded specimen. The outstanding feature is the collapse of the fibrils in the craze. There is also a marked shrinkage of the deformed area near the tip of the notch. The residual strain distributions are shown in *Figure 6*. In comparing the strain distributions in the loaded states (*Figure 3*) and the unloaded states (*Figure 6*) it is important to note the difference in the strain scale. The difference between the loaded and unloaded strains for each notch opening is shown in *Figure 7*; this is called the recovered strain. The stress associated with the recovered strain as inferred from the stress–strain curve in *Figure 1a* is shown in *Figure 8*. It is interesting to compare *Figure 8* with *Figure 4*. Both figures show the same rapid increase in stress, starting from the zero position and reaching the

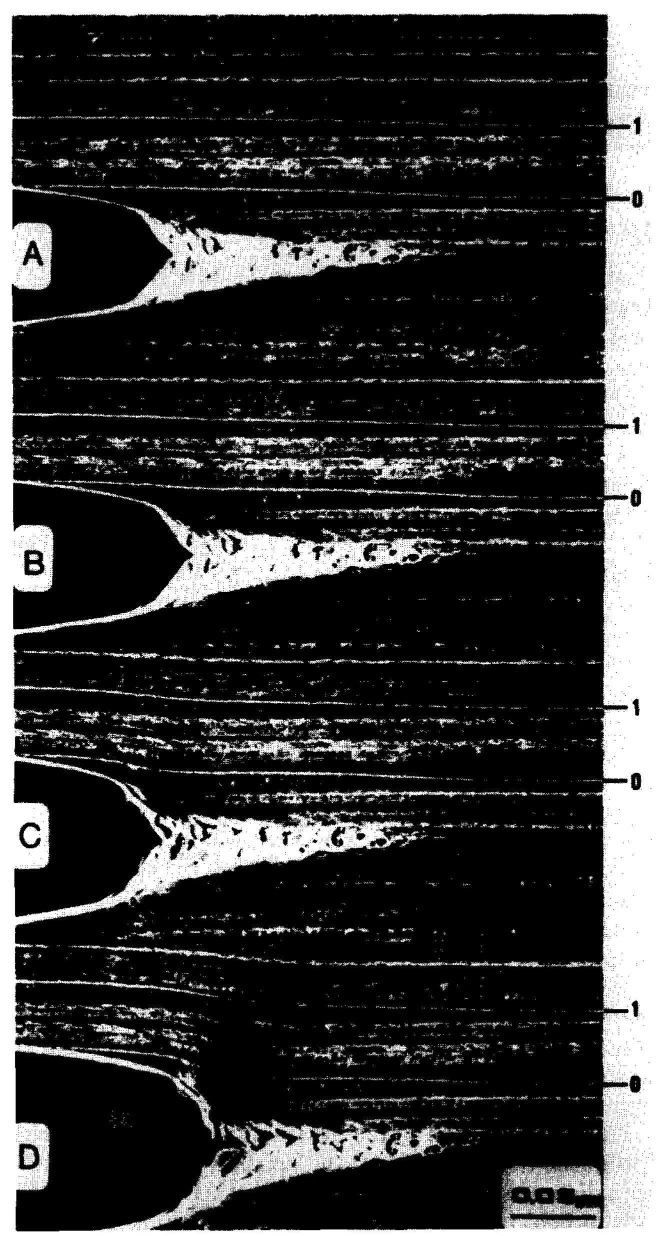

Figure 5 SEM micrographs of the unloaded state corresponding to each of the loaded states in *Figure 2*

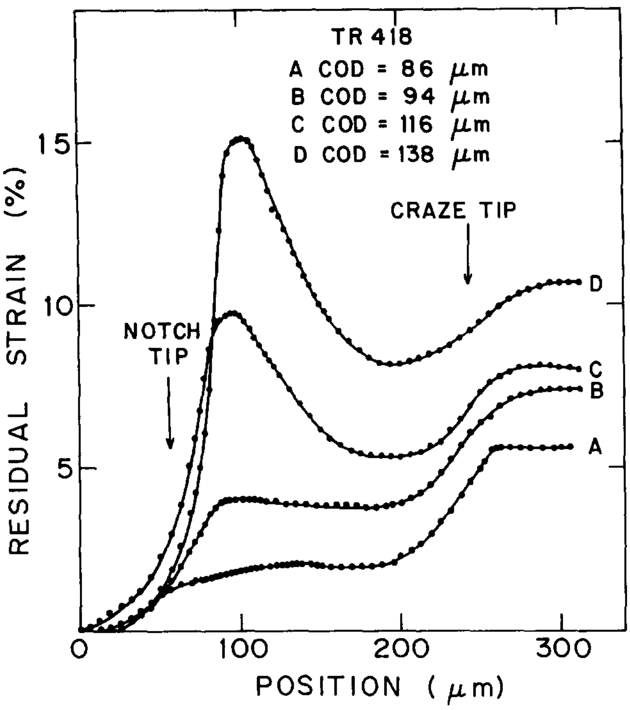

Figure 6 Same as *Figure 3* except for the unloaded states in *Figure 5*

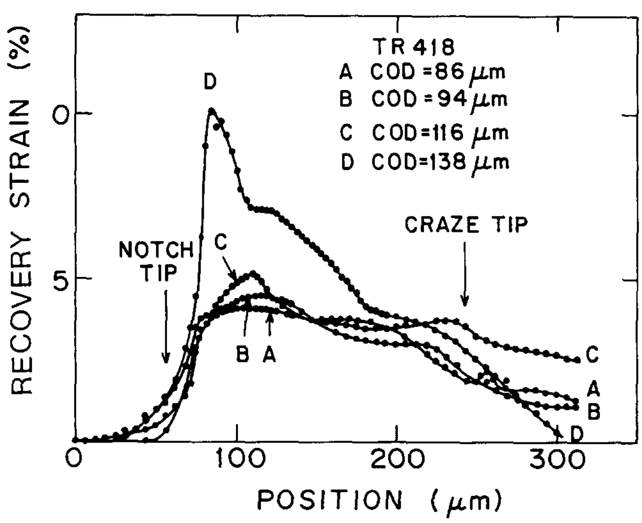

Figure 7 The recovered strain. The difference between the strain in *Figure 3* and the strain in *Figure 6*

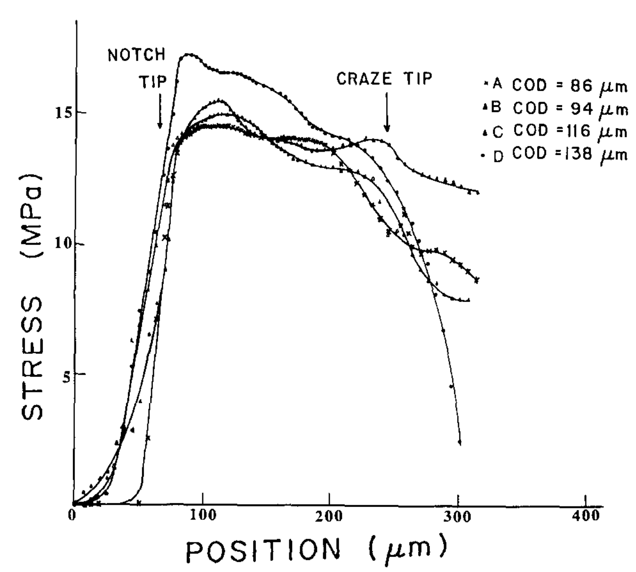

Figure 8 The stress distributions inferred from the recovered strain in *Figure 7* and from *Figure 1a*

stress level of about 15 MPa. Whereas the stress remains nearly constant from the notch tip to beyond the craze tip in *Figure 4*, the stress decreases from the notch tip to the craze tip in *Figure 8* and then more rapidly decreases beyond the craze tip. The major uncertainty in the stress distributions in *Figure 8* arises from the fact that the stress was based on the recovered strain during unloading but the stress–strain curve for obtaining the stress was determined during the loading of the material. In general, the elastic part of the stress–strain curve during loading is not the same as the elastic part during unloading, especially if the polymer has been deformed beyond the yield point.

The strain field was further explored by measuring the strain distribution at various distances from the boundary of the craze. Using another slice from the same specimen that was used in the previous section, the notch was opened at room temperature to 144 μm as compared to the notch opening of 86 μm when the craze was first formed at 80°C. The SEM micrograph of the specimen is shown in *Figure 9*. The opening of the notch to 144 μm fractured the fibrils at the base of the craze. The strain distribution at five distances from the craze boundary (*Figure 9*) are shown in *Figure 10*. Point A corresponds to the original notch tip, and the stress is now zero at this point because the neighbouring fibril had been fractured. In comparison with curve D in *Figure 3*, which also had about the same notch opening, 138 vs. 144 μm, it is seen that the sharp maximum at the position 100 μm is replaced by an appreciably smaller maximum in *Figure 10* at the same position. It is suggested that the specimen in *Figure 9* suffered more fibril fracture than the specimen in *Figure 2* with the same notch opening. As long as the fibrils near the base of the notch remain strong, then the plastic area that occurs near the notch tip can increase as the notch is opened. In *Figure 10* the strain distribution is rather constant from point B to beyond the craze tip at C.

The general level of the strain along the boundary of the craze decreases from about 17% to about 10% as the distance from the boundary varies from 46 to 144 μm (*Figure 11*). The stress level corresponding to the strains in *Figure 11*, as inferred from the stress–strain curve (*Figure 1a*), is equal to about 18 MPa and is rather

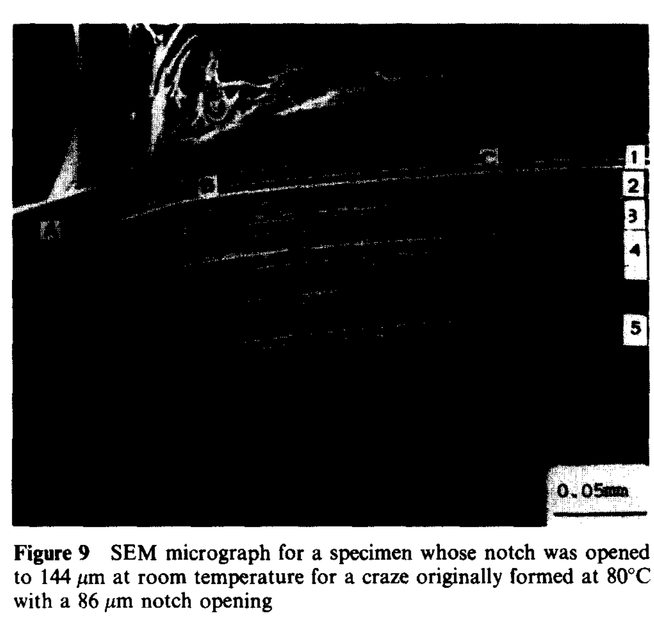

Figure 9 SEM micrograph for a specimen whose notch was opened to 144 μm at room temperature for a craze originally formed at 80°C with a 86 μm notch opening

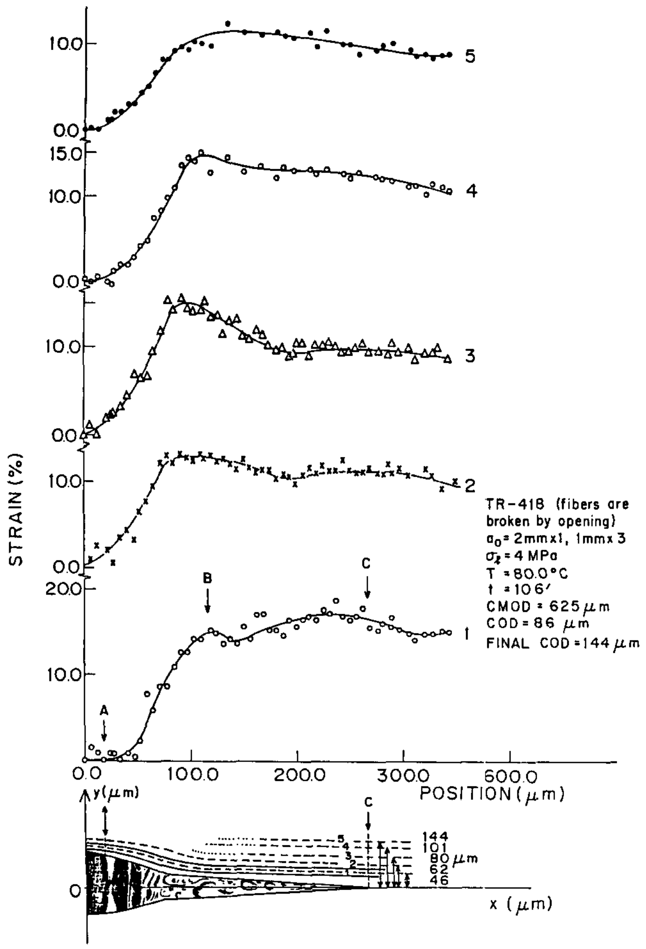

Figure 10 The strain distributions for the specimen in *Figure 9* for various distances from the boundary of the craze

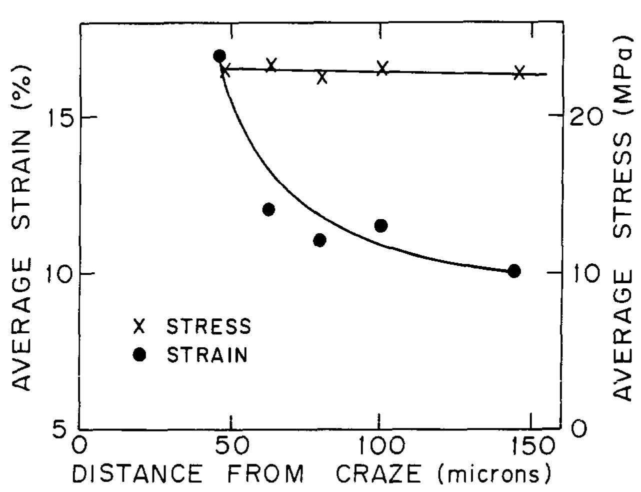

Figure 11 The change in the average level of the strain (*Figure 10*) with distance from the boundary of the craze. The stress levels were inferred from the strain using *Figure 1a*

constant over the area that was surveyed. Its variation both along the craze and away from the craze boundary is slight.

In order to determine the stress far from the notch, the applied stress was measured as a function of the notch opening. Another slice from the same specimen was subjected to increasing loads and while the notch opening was measured with a microscope. *Figure 12* shows applied stress *versus* the fraction of the initial notch opening, which was 86 μm. Before loading, the notch is already open to about 0.4 of the initial notch opening because once a craze is formed the notch cannot completely close when the load is removed, as shown in *Figure 5*. The fractional notch openings in *Figure 3* range from 1 to 1.6 and the applied stress in this range goes from 5.9 to 7.1 MPa. From *Figure 4*, the inferred stress at the furthest distance from the craze tip goes from 16.5 to 17.5 MPa over the same range of notch openings. This result suggests that the inferred stress from *Figure 4* at a point 300 μm from the notch tip does not represent the far-field stress in the specimen. Since 300 μm divided by the length of the notch (2000 μm) is small compared to unity, then it is expected that for distances of the order of 300 μm or less from the notch tip the stress is close to the yield point, as indicated by *Figures 3, 8* and *11*. In order to reach a point where the far-field stress can be measured, the point must be at a distance of about the notch length (2000 μm) from the notch tip, assuming that the stress varies as $(a_0/x)^{1/2}$ where a_0 is the notch length and x is the distance from the notch tip.

In the last phase of this study, the difference between plane stress and plane strain was investigated. In the previous sections the notch was initially loaded under plane-strain conditions at 80°C for about 100 min under 4 MPa and a subsequent thin slice from this specimen was loaded at room temperature under plane-stress conditions. In this section the notch was initially loaded at 80°C for 300 min under plane-stress conditions with a stress of 4 MPa. The scratches were formed on the specimen prior to the formation of the craze. In order to make the micrograph shown in *Figure 13*, the notch was then opened at room temperature to about the same notch opening as when the specimen was initially loaded.

The most prominent difference between the craze that was formed under plane-stress conditions (*Figure 13*) compared to the craze formed under plane-strain conditions (*Figure 2A*) is that the plane-strain craze is larger, even though it was exposed to the same stress for a shorter time at 80°C. There is no doubt that the hydrostatic component of the stress field is extremely important in determining the size of the craze. The plane-stress craze is only 22 μm long whereas the plane-strain craze is 190 μm long.

The strain distribution for the plane-stress craze is shown in *Figure 14*. The maximum is greater than for the plane-strain case (*Figure 3*, curve A). The fibrillar character of the craze is not as well developed

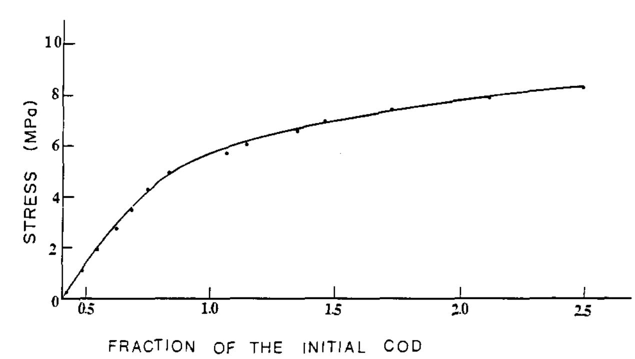

Figure 12 Applied stress *versus* the notch opening for a specimen comparable to that shown in *Figure 2*. The 1.0 fraction corresponds to a notch opening of 86 μm

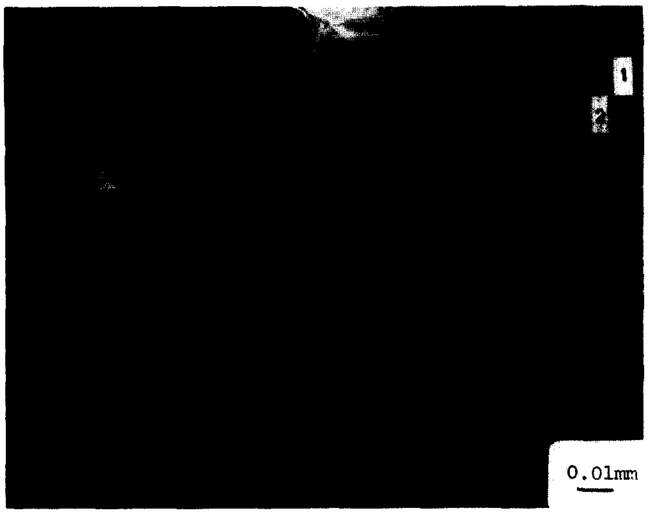

Figure 13 SEM micrograph of a craze formed at 80°C by a 4 MPa stress for 300 min under plane-stress conditions

Figure 15 Same as *Figure 13* at a higher magnification

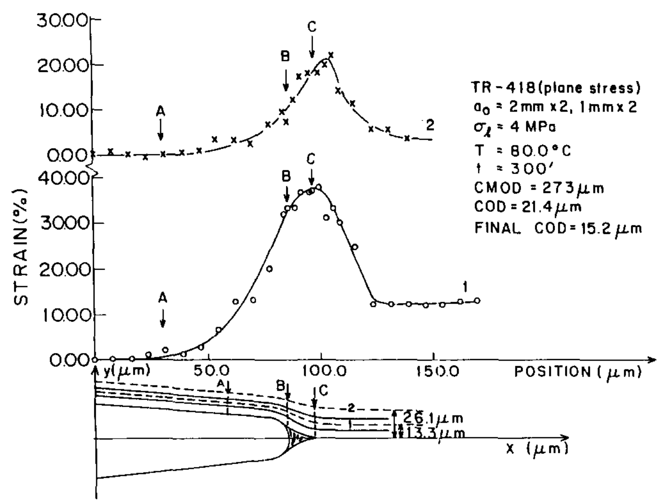

Figure 14 The strain field corresponding to *Figure 13*

(*Figure 15*) as for plane strain (*Figure 2A*). The maximum in the strain decreases rapidly with distance from the craze, going from 40 to 20%, corresponding to distances of 13 and 26 µm, respectively. The strain beyond the craze tip changes from 15 to 5%. These changes with distance from the craze are fractionally much greater than for plane strain, as exhibited by *Figure 10*.

DISCUSSION

Using a relatively new method for measuring the strain field in the neighbourhood of a notch and for inferring the stress field, four aspects were investigated:

(1) the effect of varying the notch opening;
(2) the effect of unloading the specimen after various amounts of notch opening;
(3) the effect of distance from the notch; and
(4) the difference between plane strain and plane stress.

Although the strain field is rich in some details, such as exhibiting maxima at the notch tip and/or at the craze tip, the stress field is more uniform in its behaviour. This is because the stress–strain curve from which the stress was inferred varies smoothly and exhibits only a very gentle maximum at about 12% strain.

At distances from the notch of the order of 300 µm or less, the stress field is generally constant and about equal to the yield point, except where the free surface of the notch is approached and the stress rapidly approaches zero. Whereas the applied stress is appreciably less than the yield point, the stress close to the boundary is approximately the yield point and thus in accord with the Dugdale theory. The fact that the stress–strain curve of polyethylene is highly non-linear is a most important factor in determining the stress field in the neighbourhood of a notch.

ACKNOWLEDGEMENTS

The Department of Energy and the Gas Research Institute sponsored the research. The Central Facilities of the Laboratory for Research on the Structure of Matter as supported by the National Science Foundation were most helpful.

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